From multipartite entanglement to TQFTs

Joint w/ Pavel Putrov & Abhijit Gadde

entanglement measures

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \dots \otimes \mathcal{H}_p$$

p = # of parties

β_{α_i} $\quad i = 1, \dots, p$

$\alpha_i = 1, \dots, \dim \mathcal{H}_i$

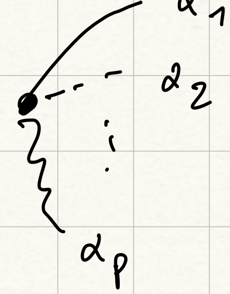
$$\Psi = \sum_{\alpha_i} \psi_{\alpha_1 \dots \alpha_p} \beta_{\alpha_1} \otimes \dots \otimes \beta_{\alpha_p}$$

to measure entanglement

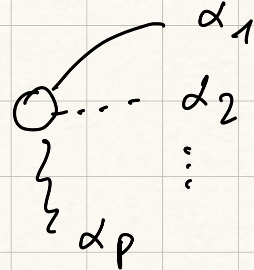
info in Ψ that is invariant under local unitary transformations

$$U = U_1 \otimes U_2 \otimes \dots \otimes U_p$$

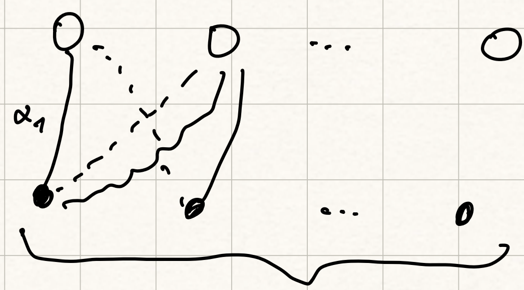
$$U_i \in \mathcal{U}(\mathcal{H}_i)$$

$\psi_{\alpha_1 \cdots \alpha_p}$ 

$\in \mathcal{H}$

$\bar{\psi}^{\alpha_1 \cdots \alpha_p}$ 

$\in \mathcal{H}^*$



R = # of REPLICAS

$\rightsquigarrow \sum_{\Gamma_p} (\Psi)$ $\qquad \Gamma_p$ bipartite graph

"entanglement measure"

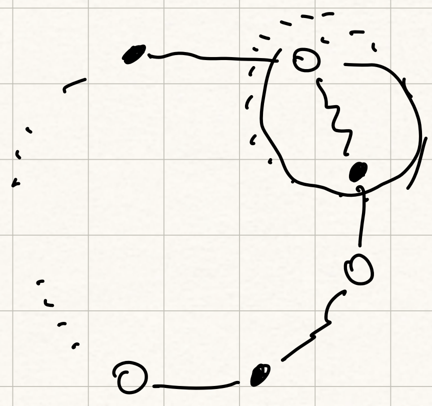
<u>example</u>

$P = 2$

$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$

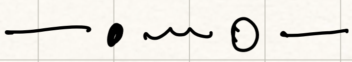
$\psi_{\alpha_1 \alpha_2}$ 

$\bar{\psi}^{\alpha_1 \alpha_2}$ 

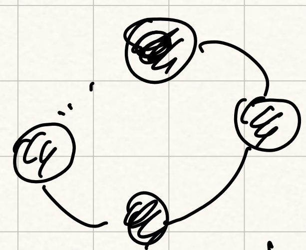


$R = \#\,\bullet$

R-th Rényi entropy



$$\rho_{\alpha_1}{}^{\alpha_1'} = \sum_{\alpha_2} \psi_{\alpha_1\alpha_2} \overline{\psi}^{\alpha_1'\alpha_2}$$



$$\mathrm{Tr}\, \rho^R$$

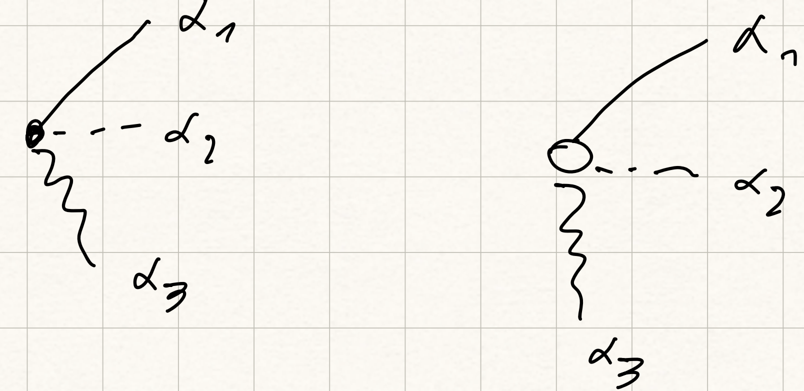
$$\lim_{R\to 1} \frac{1}{1-R} \log \mathrm{tr}\, \rho^R$$

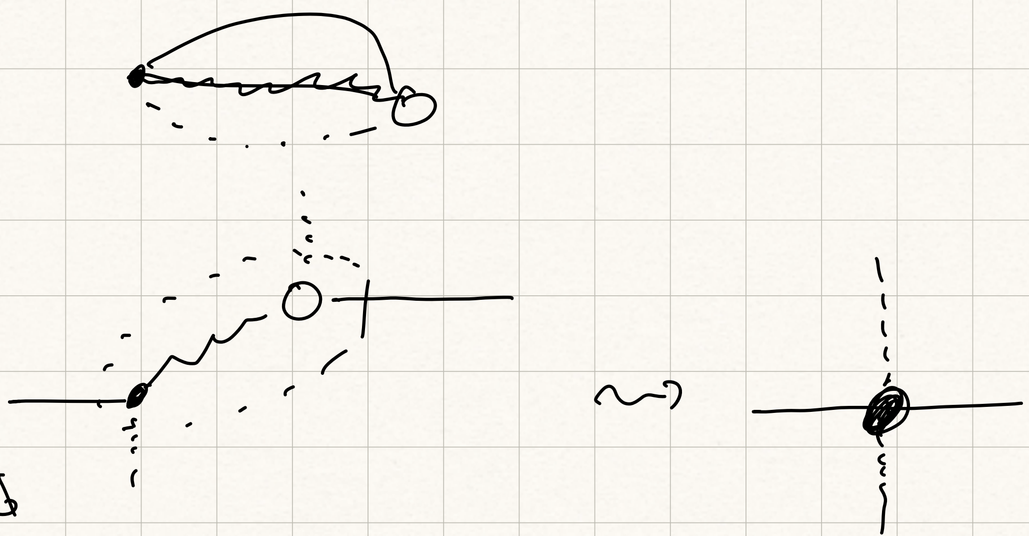
$\rightsquigarrow$ von Neumann entropy ρ

multipartite entanglement

one could impose / ask further properties (vanish on prod. state, monotonicity ...)

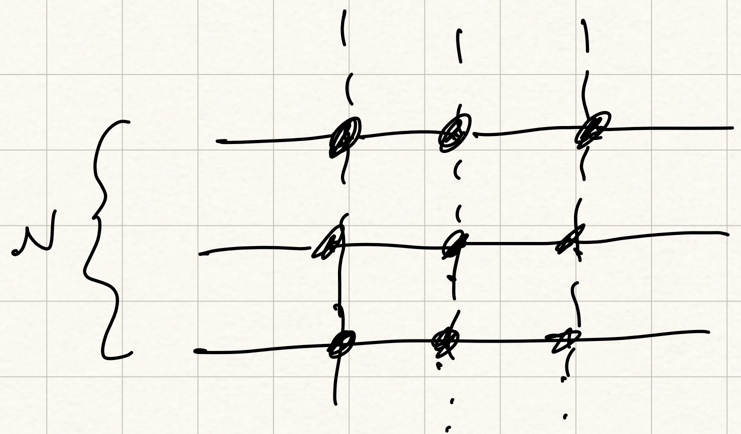
$P = 3$





RMK

there we generalized vN type p-partite entropies



reduced density "tensor"

$\rightsquigarrow Z_N^{(3)}$

$$S^{(3)} = \frac{1}{1-N} \lim_{N\to 1} \log Z_N^{(3)}$$

good characterization of genuine

N-partite entanglement ⇝ OPEN ?

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ground state Ψ

$\Sigma_p(\Psi)$ $\overset{?}{\longleftrightarrow}$ TQFT

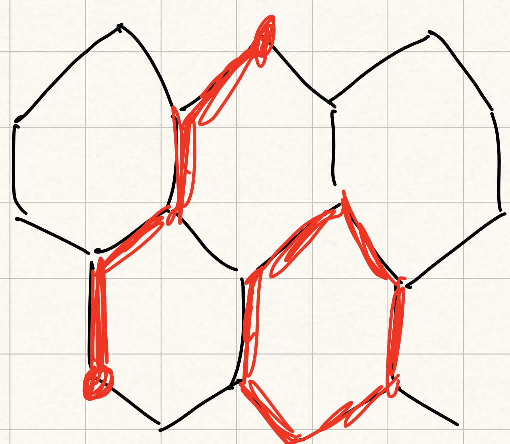
gapped lattice models

→ g.s. are described by TQFT

string-net models Levin-Wen

⇝ kind of Turaev-Viro

ground states are known



string net configurations

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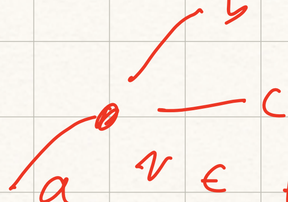
graphical calculus of spherical fusion CAT $\mathcal{C}$

in the IR

→ Turaev Viro TQFT for $\mathcal{C}$



$v \in \mathrm{Hom}(a, b \otimes c)$

* Kirillov Jr & Balsam ~ 2010

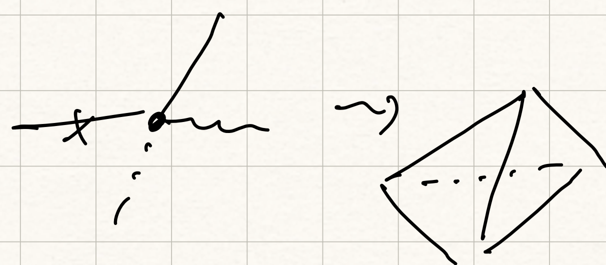
ground state Ψ_{LW}

$$\boxed{\log \Sigma_{\Gamma_4}(\Psi_{LW}) = \log Z_{TV}(M^{(3)}_{\Gamma_4}) + O(e^{-cL})}$$

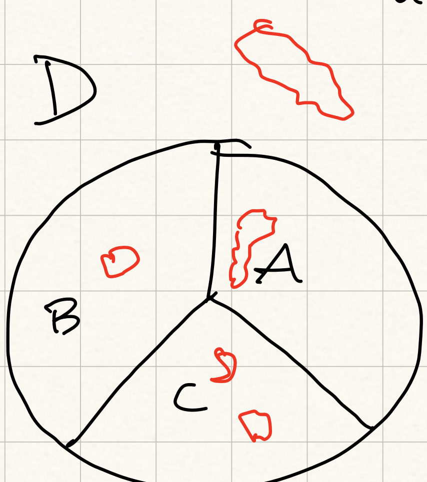
Graphically encoded 3-manifolds

generalizes

Kitaev-Preskill / Levin-Wen / 2006



planar hexagonal lattice divided into 4 big regions



⇝ 4-partite entanglement measures

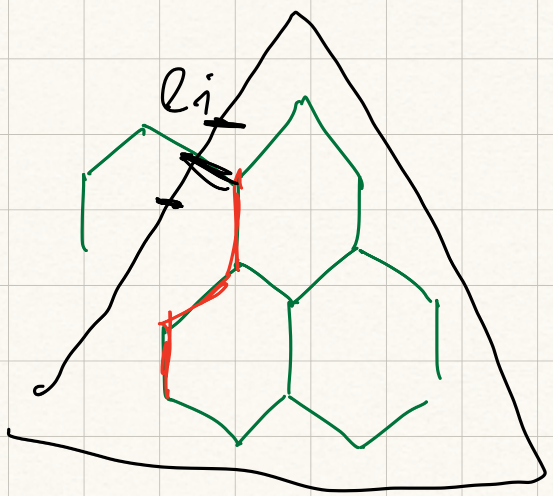
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C \otimes \mathcal{H}_D$$

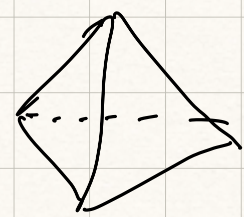
strings don't go to ∞

$\rightsquigarrow$ topologically it's on S^2

S

boundary





$\ell_i \in \mathrm{Irr}(\mathcal{C})$

$$\mathcal{H} = \mathrm{Hom}_{\mathcal{C}}(1, \ell_1 \otimes \cdots \otimes \ell_m)$$

Ψ_{cw} can construct it explicitly using Kirillov Jr work

we can show ⊛ provided

$$\sum_{\ell \in \mathrm{Irr}(\mathcal{C})^L} N^{\hat{e}}_{\ell_1 \ldots \ell_L} \prod_{i=1}^{L} d_{\ell_i}^{n}$$

$$= \frac{\left(\sum_{e \in \mathrm{Irr}(\mathcal{C})} d_e^{\,n+1}\right)^L d_{\hat{e}}}{\mathcal{D}^2 = \sum_{e \in \mathrm{Irr}(\mathcal{C})} d_e^2} \left(1 + O(e^{-cL})\right)$$

$L \gg 1$

$c > 0$

$$N^{\hat{e}}_{e_1 \dots e_L} = \dim \mathrm{Hom}_{\mathcal{C}}(e_1 \otimes \dots e_L, \hat{e})$$

$n = 1$ is well-known

$$\sum_{e \in \mathrm{Irr}(\mathcal{C})^L} N^{\hat{e}}_{e_1 \dots e_L} \prod_{i=1}^{L} d_{e_i}$$

$$= \mathcal{D}^{2L-2} d_{\hat{e}}$$

$n > 1$ Povet can prove it when $\mathcal{C}$ ~~is~~ is an MTC

In general we expect

d-dim'l TQFT on $M^d_{\Gamma_{d+1}}$	$\longmapsto$	$(d+1)$-partite entanglement measures $\mathcal{E}_{\Gamma_{d+1}}(\Psi_0)$

d+1

gluing Δ^{d+1}
symplexes

Questions

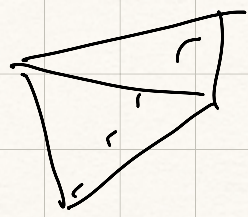
1.) what do we need to
completely characterize
a TQFT?

having all partition
functions is it enough?

2.) what is special of
the $\Gamma \to$ manifolds
(genuine multipartite
entanglement?)

3.) when we classify phases
and orders
$\to$ should we ask we
retain info about
entanglement?

Graph-encoded manifolds



Δ^4